

Constraints on Cardassian Expansion

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ABSTRACT

High redshift supernovae and Cosmic Microwave Background data are used to constrain the Cardassian expansion model (Freese & Lewis, 2002), a cosmology in which a modification to the Friedmann equation gives rise to a flat, matter-dominated Universe which is undergoing a phase of accelerated expansion. In particular, the precision of the positions of the Doppler peaks in the CMB angular power spectrum provided by the Wilkinson Microwave Anisotropy Probe (WMAP) tightly constrains the cosmology. The available parameter space is further constrained by various high redshift supernova datasets taken from Tonry et al. (2003), a sample of 230 supernovae collated from the literature, in which fits to the distance and extinction have been recomputed where possible and a consistent zero-point has been applied. In addition, the Cardassian model can also be loosely constrained by inferred upper limits on the epoch at which the Cardassian term in the modified Friedmann equation begins to dominate the expansion (z_{eq}). Using these methods, a Cardassian cosmology is constrained at the 2σ level to $0.19 < \Omega_m \lesssim 0.26$, $0.01 < n < 0.24$ for the Cardassian expansion parameter, n , and $0.42 < z_{eq} < 0.89$, in contradiction to the previous constraints of Sen & Sen (2003a). There is also a large discrepancy between the 1σ confidence regions defined by the CMB and tightest Supernova constraints, with the CMB data favouring a low- Ω_m , high- n Cardassian cosmology and $z_{eq} > 1$, as opposed to the Supernova data which supports a high- Ω_m , low- n cosmology.

1 INTRODUCTION

Over the last ten years, the m - z relation below $z \approx 1$ has been constrained through observations of distant type Ia supernovae, and indicates that the Universe is currently in a phase of accelerated expansion (e.g. Perlmutter et al. 1997; Riess et al. 1998). This is supported by recent measurements of the Cosmic Microwave Background angular power spectrum and the galaxy power spectrum. The standard model provides an interpretation of these results by proposing that the Universe consists of only ~ 4 per cent baryonic matter with the remainder in the form of Cold Dark Matter and a dark energy component which drives the expansion. This cosmology provides a good fit to the Wilkinson Microwave Anisotropy Probe (WMAP) CMB power spectrum and high redshift supernova data, and is consistent with local large-scale structure and cluster baryon fraction measurements of the matter density. This cosmology is slightly problematic however as it invokes the use of two unobserved forms of energy. While the evidence for Cold Dark Matter is considerable, evidence for the existence of a dark energy component remains indirect.

An interesting alternative to the standard model of cosmology recently proposed by Freese & Lewis (2002) invokes an additional term in the Friedmann equation, proportional to ρ^n , which results in a Universe which is consistent with the recent evidence for an increasing expansion rate, but is

both flat and matter dominated. The need for a dark energy component is therefore removed, and the expansion is driven solely by the new ρ^n term in the modified Friedmann equation. The theoretical motivation for this Cardassian model of cosmology (see Freese & Lewis 2002) is fairly speculative however, and the implications for Einstein's equations of General Relativity are currently undetermined.

In order to constrain the Cardassian model, it is critical that the observational data used do not assume a prior cosmology. The simplest constraints can therefore be imposed by the CMB angular power spectrum, high redshift supernova data and local large-scale structure, since in a Cardassian cosmology these observables depend on Ω_m , the Cardassian parameter n , and the redshift at which the Cardassian term in the modified Friedmann equation begins to dominate, z_{eq} .

Previously, Sen & Sen (2003a) used predictions for the locations of the first and third Doppler peaks in the CMB power spectrum for a Cardassian cosmology, and Archeops and Boomerang data to constrain possible values for Ω_m and n . Supernova data from the Calan-Tololo project and the Supernova Cosmology Project (Perlmutter et al. 1997) were also used to constrain these parameters, since the modification of the luminosity distance in a Cardassian cosmology alters the predicted apparent magnitude for supernovae. These two constraints restricted the parameters to $0.31 \lesssim n \lesssim 0.44$ and $0.13 \lesssim \Omega_m \lesssim 0.23$. Zhu & Fujimoto (2003)

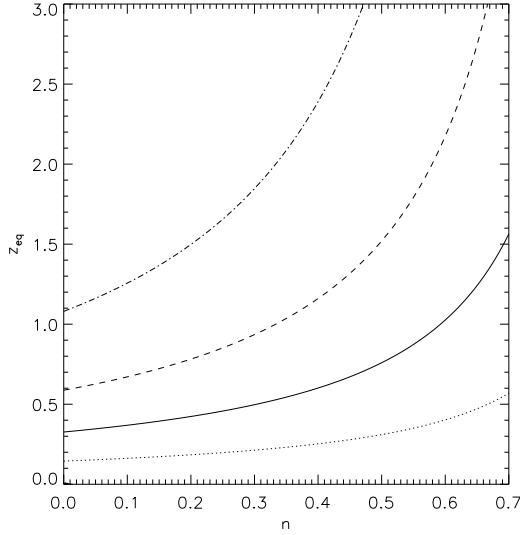


Figure 1. Contours of Ω_m with the Cardassian parameters n and z_{eq} . $\Omega_m=0.1$ (dot-dashed), $\Omega_m=0.2$ (dashed), $\Omega_m=0.3$ (solid) and $\Omega_m=0.4$ (dotted) tracks are shown.

also used these supernova data to constrain the Cardassian model, and found best fit parameters of $n=-1.33$ and $z_{eq}=0.43$ assuming $\Omega_m=0.3$. However, it is not clear how the fit is affected for different values of Ω_m . More recently Sen & Sen (2003b) used WMAP and Boomerang measurements of the locations for the first, second and third Doppler peaks to further constrain the parameter space.

In this paper, the parameter space is constrained using the WMAP and Boomerang measurements for the locations of the first, second and third Doppler peaks in the CMB angular power spectrum used in Sen & Sen (2003b), and various high redshift supernova datasets taken from Tonry et al. (2003). An orthogonal constraint is also applied using an assumed upper limit of $z_{eq}=1$; this value is used in order to be consistent with local large-scale structure (Freese & Lewis, 2002). In Section 2, the Cardassian model is outlined. The supernova analysis method and the resulting constraints for various samples are presented in Section 3, along with the CMB constraints and the additional constraint imposed by the inferred upper limit on z_{eq} . The discussion and conclusions follow in Section 4.

2 THE CARDASSIAN MODEL

An additional term is added to the Friedmann equation such that

$$H^2 = A\rho + B\rho^n \quad (1)$$

where $A = \frac{8\pi G}{3}$, H is the Hubble constant, ρ is the energy density and n is a parameter of the Cardassian model. When the second term dominates, the scale factor $a \propto t^{\frac{2}{3n}}$, gives rise to accelerated expansion for $n < \frac{2}{3}$. In order to determine B , it is convenient to equate the first and second terms

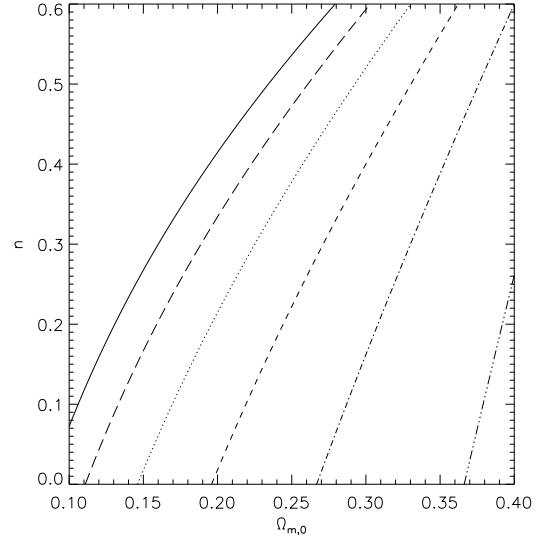


Figure 2. Contours of z_{eq} with the Cardassian parameter, n , and Ω_m . Contours for $z_{eq}=1.2$ (solid), $z_{eq}=1.0$ (large-dashed), $z_{eq}=0.8$ (dotted), $z_{eq}=0.6$ (small-dashed), $z_{eq}=0.4$ (dot-dashed) and $z_{eq}=0.2$ (dot-dot-dashed) are shown.

at z_{eq} , the epoch at which the Cardassian term begins to dominate. Since $\rho_m = \rho_{m,0}(1+z)^3$

$$B = \frac{8\pi G}{3} [\rho_{m,0}(1+z_{eq})^3]^{1-n} \quad (2)$$

assuming the radiation contribution to the energy density is negligible. This expression gives rise to a modification in the critical energy density such that in a Cardassian cosmology

$$\rho_{crit} = F(n, z_{eq}) \times \rho_{crit,old} \quad (3)$$

where

$$F(n, z_{eq}) = [1 + (1 + z_{eq})^{3(1-n)}]^{-1} \quad (4)$$

which for a flat cosmology is equivalent to Ω_m . Hence, this modification to the critical energy density is consistent with a flat, matter dominated cosmology. Fig. 1 shows the variation of Ω_m ($\equiv F(n, z_{eq})$) with n and z_{eq} . Equivalently, Fig. 2 shows the variation of z_{eq} with Ω_m and n .

3 OBSERVATIONAL CONSTRAINTS

3.1 Supernova Data

In order to constrain the cosmological parameters of the Cardassian model, firstly, high redshift supernova data is used. Since altering the cosmology modifies the luminosity distance, the Cardassian model can be constrained by comparing empirical data with predicted values for the apparent magnitudes of standard candles at known redshifts. The relation between apparent and absolute magnitude can be written in the form (Perlmutter et al. 1997)

$$m_B = \mathcal{M}_B + 5 \log \mathcal{D}_L \quad (5)$$

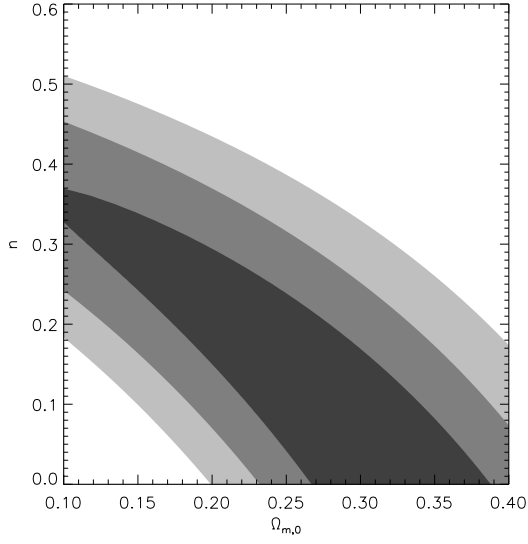


Figure 3. Constraints from all 230 supernovae listed in Tonry et al. (2003) on Ω_m and the Cardassian parameter n . The 1σ (darkest shade), 2σ and 3σ confidence regions are shown.

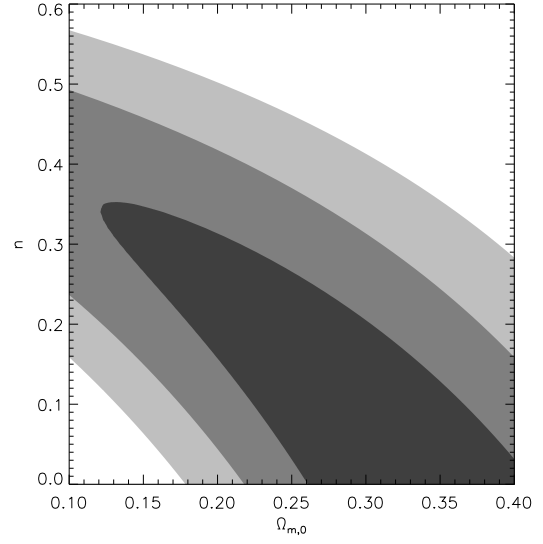


Figure 5. Constraints from the 130 supernovae published by the High- z Supernova Search Team (taken from Tonry et al. (2003)) on Ω_m and the Cardassian parameter n . The confidence regions are shown as before.

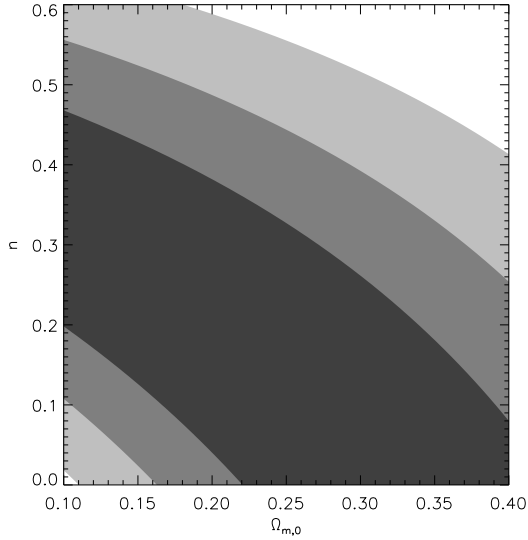


Figure 4. Constraints from the 54 supernovae published in Perlmutter et al. (1997) in Fit C, including supernovae from the Calan-Tololo Survey and the Supernova Cosmology Project (taken from Tonry et al. (2003)), on Ω_m and the Cardassian parameter n . The confidence regions are shown as before.

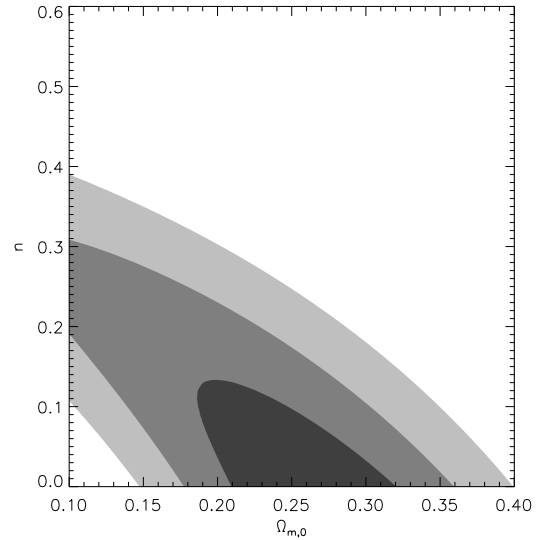


Figure 6. Constraints from the 172 supernovae (for which a redshift limit of $z > 0.01$ and an extinction limit of $A_V < 0.5$ mag. were applied where possible) on Ω_m and the Cardassian parameter n . The confidence regions are shown as before.

where $\mathcal{M}_B = M_B - 5 \log H_0 + 25$ is the Hubble constant-free absolute B -band magnitude, and is determined empirically for type Ia supernovae as $\mathcal{M}_B = -3.32 \pm 0.05$ (Perlmutter et al. 1997) from the intercept on the Hubble diagram. $\mathcal{D}_L = H_0 d_L$ is the Hubble constant-free luminosity distance. The luminosity distance is given by

$$d_L(z) = \frac{c}{H_0} (1+z) \int_0^z \frac{dz}{E(z)} \quad (6)$$

where $E(z)$ in a Cardassian cosmology is given by (Freese & Lewis, 2002)

$$E^2(z) = \Omega_m (1+z)^3 + (1 - \Omega_m) (1+z)^{3n} \quad (7)$$

Equations 5, 6 and 7 determine a theoretical prediction for the apparent magnitude ($m_B(\Omega_m, n)$) of a supernova at a particular redshift. Hence, Ω_m and n are constrained through a determination of $\chi^2 - \chi_{min}^2$ where

$$\chi^2 = \sum \frac{(m_B(\Omega_m, n) - m_{B_i})^2}{\sigma_{m_{B_i}}^2} \quad (8)$$

where $\sigma_{m_{B_i}}$ is determined by quadrature of the errors on $\log \mathcal{D}_L$ and \mathcal{M}_B . For a more robust determination of the errors, the uncertainty in \mathcal{M}_B and the distance should be integrated over the entire parameter space, and an error on the redshift could also be included in the determination of χ^2 . However, these makes little difference to the resulting confidence regions and the determination of the errors described above is adequate for this analysis.

The observational data is taken from Tonry et al. (2003), which presents redshift and distance information for 230 supernovae compiled from the literature and eight new supernovae from the High- z Supernova Search Team. Since the techniques for analysing the observational data vary between individual samples of supernovae, the authors have attempted to recompute the extinction estimates and the distance fitting using the methods of Riess et al. (1998), Jha et al. (2003) and Tonry et al. where possible, as well as apply a consistent zero-point for the local distance calibration. A complete description of this procedure is given in Tonry et al. (2003).

Four supernova subsamples are selected from the 230 supernovae listed in Tonry et al. (2003). Firstly, the entire sample of 230 supernovae is used; the resulting 1σ , 2σ and 3σ constraints are shown in Fig. 3. Secondly, the 54 supernovae included in Fit C in Perlmutter et al. (1997) taken from the Calan-Tololo Survey and the Supernova Cosmology Project are selected (see Fig. 4). Thirdly, the 130 supernovae originally published by the High- z Supernova Search Team are used to constrain the parameter space (see Fig. 5). Finally, a redshift cut of $z > 0.01$ and an extinction cut of $A_V > 0.5$ mag. is applied where possible, as in Tonry et al. (2003), in order to remove objects where an uncertain effect is apparent from either local peculiar velocities or host galaxy extinction; the constraints arising from the resulting 172 supernovae are shown in Fig. 6.

3.2 The Cosmic Microwave Background

In order to further constrain the available parameter space of the Cardassian model, empirical measurements for the locations of the first three Doppler peaks in the CMB angular power spectrum are compared with predicted values in a Cardassian cosmology. From Doran et al. (2001), the acoustic scale can be defined in terms of the conformal time

$$l_A = \pi \frac{\tau_0 - \tau_{ls}}{c_s \tau_{ls}} \quad (9)$$

where τ_0 and τ_{ls} are the conformal time at the present day and at last scattering respectively. c_s defines the mean speed of sound before last scattering and is constant for a particular $\frac{\rho_b}{\rho_{rad}}$ and is taken to be 0.52 (Doran et al. 2001). The definition derived by Sen & Sen (2003a) for this expression of l_A in a Cardassian cosmology is

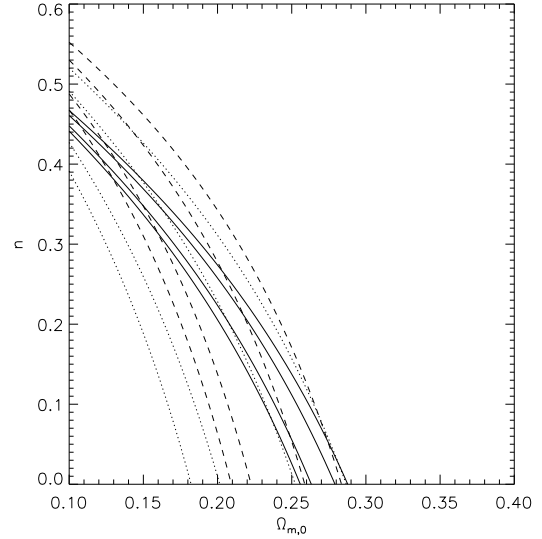


Figure 7. Constraints on Ω_m and the Cardassian parameter, n , from the location of the first (solid) and second (dotted) Doppler peaks in the WMAP angular power spectrum, and the third (dashed) Doppler peak from Boomerang. The 1σ and 2σ contours are shown in each case.

$$l_A = \frac{\pi}{c_s} \left(\frac{\int_0^{a_0} \frac{da}{X(a)}}{\int_0^{a_{ls}} \frac{da}{X(a)}} - 1 \right) \quad (10)$$

where $a_0=1$, $a_{ls}=1100^{-1}$ and

$$X(a) = \left(a + a^{4-3n} \left(\frac{1 - \Omega_{m,0}}{\Omega_{m,0}} \right) \right)^{\frac{1}{2}} \quad (11)$$

For a spectral index of $n_s=1$, the locations of the Doppler peaks are related to the acoustic scale by the relation

$$l_m = l_A(m - \phi_m) \quad (12)$$

where ϕ_m is a phase shift arising from driving and dissipative effects in the photon-baryon fluid before last scattering, and is weakly dependent on Ω_m , $\Omega_b h^2$ and n_s (see Doran & Lilley 2002). The expressions for ϕ_m for the first three acoustic peaks are given in Sen & Sen (2003b). Hence for assumed values of $\Omega_b h^2$ and n_s , the locations of the Doppler peaks in a Cardassian cosmology can be predicted for a particular Ω_m and n . In the following analysis, values of $\Omega_b h^2=0.02$ (e.g. Burles et al. 2001; Pettini & Bowen 2001; Kirkman et al. 2003) and $n_s=1.0$ are used, the effects of which are investigated in Sen & Sen (2003b).

The WMAP value for the locations of the first and second Doppler peaks are given as $l_1=220.1 \pm 0.8$ and $l_2=546 \pm 10$ by Hinshaw et al. (2003) from Gaussian and hyperbolic fits to the power spectrum. The location of the third peak is given as $l_3=825^{+10}_{-13}$ (Hu et al. 2001). The resulting 1σ and 2σ constraints arising from each peak location are shown in Fig. 7.

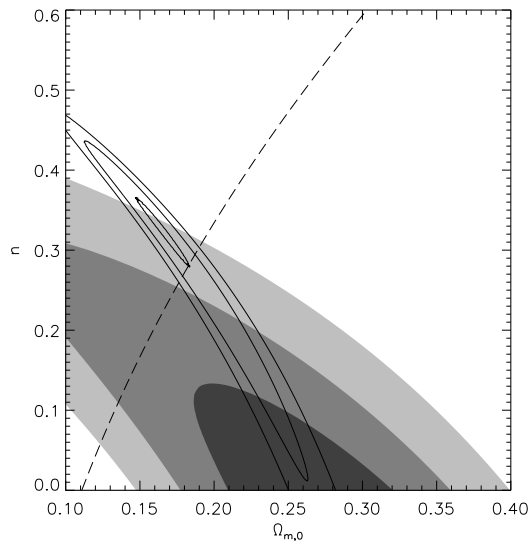


Figure 8. Here the constraints from the CMB data, the 172 supernovae sample (with $A_V < 0.5$ mag. and $z > 0.01$), and the upper limit on z_{eq} are shown. The 1σ , 2σ and 3σ confidence regions from the first three Doppler peaks have been combined and are indicated by solid contours. The supernova confidence regions are shown as in Fig. 6. The $z_{eq}=1.0$ track is indicated as a large-dashed contour as in Fig. 2.

4 DISCUSSION & CONCLUSIONS

The Cardassian expansion model utilises an additional term in the Friedmann equation, proportional to ρ^n , such that the Universe is flat, matter-dominated and in a phase of accelerated expansion. The need for a dark energy component is therefore removed, as the expansion observed in distant supernovae is driven solely by this additional component. In order to constrain the parameters of the Cardassian model n , Ω_m and z_{eq} (the epoch at which the Cardassian term begins to dominate the expansion), it is necessary to select observational data free from any cosmological priors.

In this paper, various samples of type Ia supernova data from Tonry et al. (2003) were used to constrain Ω_m and n . Fig. 4 shows the 1σ , 2σ and 3σ confidence levels derived from the 54 supernovae included in Fit C of Perlmutter et al. (1997). This is in general agreement with the constraints imposed by Sen & Sen (2003a) for the same sample of supernovae, although the constraints are slightly broader, perhaps due to their use of contouring for absolute values of χ^2 . Since the errors are uncertain, more robust confidence levels can be derived when contouring around a minimum value of χ^2 as used for the supernova data in this paper. Sen & Sen (2003a) also use a different method of analysis in which a different value of \mathcal{M}_B is used with an uncertain associated error.

The tightest constraints from the supernova data are provided by the 172 supernovae sample (Fig. 6) for which an extinction limit of $A_V=0.5$ and a redshift cut of $z > 0.01$ have been applied in order to remove objects with potentially large effects from peculiar velocities at low redshift and large host galaxy extinctions. The resulting 1σ constraints

favour a value of $0.19 < \Omega_m < 0.32$ and $n < 0.13$. However, the available parameter space within the 3σ confidence region is large, although this represents a significant improvement over previous constraints.

To constrain the available parameter space further, WMAP (Hinshaw et al. 2003) and Boomerang (Hu et al. 2001) measurements for the first three Doppler peaks in the CMB angular power spectrum are compared to the associated Cardassian prediction as in Sen & Sen (2003b) (Fig. 7). These three constraints are in remarkably good agreement at $\Omega_m \sim 0.15$, but begin to diverge at the values of Ω_m and n suggested by the 172 supernovae sample.

In Fig. 8, the combined constraints from the first three Doppler peaks and the 172 supernovae sample are combined. Also included is the $z_{eq}=1.0$ track which represents an approximate upper limit arising from the effect of the Cardassian term in the modified Friedmann equation on local large-scale structure (Freese & Lewis, 2002). The 1σ constraint imposed by the combined CMB data indicates values of $0.15 \lesssim \Omega_m \lesssim 0.18$, $0.28 < n < 0.36$ and $z_{eq} > 1.0$. However, there is good agreement at the 2σ level with constraints of $0.19 < \Omega_m \lesssim 0.26$, $0.01 < n < 0.24$ and $0.42 < z_{eq} < 0.89$. This is consistent with the loose constraint of $z_{eq} < 1.0$ imposed by local large-scale structure. However, it is in contradiction to the constraints derived by Sen & Sen (2003) of $0.13 \lesssim \Omega_m \lesssim 0.23$ and $0.31 < n < 0.44$.

In conclusion, high redshift supernova data and recent measurements of the CMB power spectrum provide relatively tight constraints on the available parameter space of the Cardassian expansion model. There is a large discrepancy between the CMB and supernova 1σ confidence regions, with the CMB 1σ constraint implying a parameter space which is potentially at odds with local large-scale structure and is rejected by the supernova data at the $>2\sigma$ level. However, there is good agreement at the 2σ level, with the associated confidence regions limiting $0.19 < \Omega_m \lesssim 0.26$, $0.01 < n < 0.24$ and $0.42 < z_{eq} < 0.89$.

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